

# A HOUSE OF YOUR OWN:

## THE MATHEMATICS OF THE AMERICAN DREAM PART II

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**Description:** Given materials drawn from community resources, students choose the most appropriate mortgage program to finance a new home.

**Keywords:** House buying, home mortgages, simple interest amortized loans, amortization schedule, financial mathematics.

**Objectives:** To engage students with hands-on experience with some of the calculations and analysis involved in buying a home; to provide motivation for various topics in financial mathematics; to convince students of the relevance of mathematics in their daily lives.

**Prerequisites:** Organizational ability, attention to detail, and comfort with basic algebraic concepts such as order of operation and properties of exponents.

**Course level:** Any course, such as algebra II, pre-calculus, financial mathematics, in which students have or develop an understanding of basic algebraic methods, or any course that develops basic formulas for simple and compound interest, and amortized loans. This is a good capstone project for the financial component of such a course.

**Technology used:** At least a calculator with a memory function and exponentiation key, and access to an amortization schedule generator, eg. on the Internet. Spreadsheet software optional.

**Required equipment/materials:** Area real estate listings, current mortgage rates, local property tax information.

**Time estimates:** *Instructor preparation:* Minimal for project provided here, 2 to 6 hours for a customized project using local information. *Class presentation:* Minimal in a class that already includes basic finance formulas, several class periods to develop the formulas as ancillary material. *Student work outside of class:* About 8 to 14 hours. *Fielding questions:* Average of about 30 minutes per group. *Grading:* Roughly 20 minutes per project submitted.

**NCTM standards addressed:** *Content standards:* number and operations, algebra.

*Process standards:* problem solving, communication, connections, representation.

## PROJECT DESCRIPTION

Surely everyone has some corner of his or her heart reserved for a dream home. It may be no more than a vague notion of someday having a “place of one’s own”. It may be a fully imagined three-bedroom ranch, with even the carpets and countertops already selected. But, to become a reality, this dream home will likely cost thousands, *hundreds* of thousands, of dollars. For a \$120,000 home, a borrower will pay more than \$360,000 over the life of thirty-year mortgage at 9.5%. Then there are taxes, mortgage insurance, points, fees, and more. Once students realize this, it is a short step to convincing them that it is worth studying the mathematics involved in buying a house. Even adult learners, (perhaps *especially* adult learners) who have been through the process of buying a house already, are eager to learn this material.

This article includes a classroom-ready project for experiencing some of the computations and analysis involved in choosing a fixed-rate home mortgage. We collected information from area banks, real estate guides, and the town office to make the project as realistic as possible. This is a substantial project which requires students to demonstrate thorough mastery of basic financial concepts and formulas, proficiency with a calculator or software program, and analytic ability in a real-life context.

Although the house advertised below comes from our local area, it is easy to modify the project for any location or price range. A house/condominium/apartment can be found in a local real estate guide, current mortgage rates can be found at any bank, and tax information can be obtained through a telephone call to the town office. If a spreadsheet is used to compute the solutions, these changes, as well as yearly updates, are

easy to implement since only the raw data needs to be entered in the appropriate cells to generate answers for the whole project.

The mathematics of house buying are substantial, but appropriate to any course requiring basic algebra skills. For this project, students need to know how to determine the down payment of a loan, calculate the monthly payment of a simple interest amortized loan, and use the APR to estimate prepaid finance charges. The respective equations for this are:

$A = p(C - D) + D$ , where  $A$  is the amount of cash available for points and down payment,  $p$  is the points, and  $C$  is the cost of the house;

$P(1+i)^n = \text{payment} \frac{(1+i)^n - 1}{i}$ , where  $P$  is the principal,  $i$  is the periodic rate, and

$n$  is the number of payments; and

$(P - \text{fees})(1+i)^n = \text{payment} \frac{(1+i)^n - 1}{i}$ , where  $P$  is the principal,  $i$  is the annual

percentage rate,  $n$  is the number of payments, and  $\text{payment}$  is the amount of the monthly principal and interest payments.

In the last equation, solving for  $\text{fees}$  gives an estimate of the prepaid finance charges.

This is a minor part of the project and can easily be omitted.

Other mathematical activities involved in this house buying project include calculating points, understanding maximum LTV (Loan to Value) and its relation to a minimum down payment, becoming familiar with escrow of mortgage insurance and property taxes, computing property taxes from the fair market value of a home, knowing when to use the APR and when to use the rate, and preparing a long term amortization schedule.

In our own courses, most students simply use calculators for all parts of the project except the amortization schedule, which they prepare using a software package. Students can create an abbreviated (3 to 6 month) amortization schedule using a calculator. They often gain a better understanding of the mechanics of a mortgage (realizing why it is called a *simple interest* amortized loan, for example) by working out these details. However, the tedium of computing a lengthy schedule by hand quickly provides great motivation for learning to use a spreadsheet or other software. In fact, although this project can be done using only a calculator, it is especially suited for courses which incorporate some kind of spreadsheet program.

Student response to this project in our courses has been overwhelmingly positive. They are motivated to do substantial mathematics and feel that the experience is valuable. Their enthusiasm stems from being convinced that what they have learned will help them realize their dream of someday having a home of their own.

The classroom materials and solutions for this project are below.

## THE HOUSE BUYING PROJECT

You have decided to buy the house advertised below. The house is located in Stoneville, where property appraisals were redone fairly recently, so that properties are appraised at 86% of market value. The tax rate in Stoneville is \$1.93.



**Idyllic Retreat**

Rural Stoneville, convenient to mountains and lake. 5.4 acres with beautiful pond and stream. 2,500 SF, 3BRs, 2-1/2 baths. Finished basement, studio/office. Chef's kitchen. Motivated sellers. \$129,500.

You have saved \$12,000. \$3,500 of that amount is needed to pay for prorations, taxes, lawyer's and inspector's fees, appraisal fee, utilities hook ups, mortgage insurance prepayment, and other charges. The remainder of your savings will pay for your down payment and any points.

Since you will have to borrow more than 80% of the value of the house, you will probably have to pay mortgage insurance. Assume this costs \$75 per month. In addition, you will need to use the local tax rate and percent of fair market value for the town to estimate your yearly property tax. Divide this amount by twelve to find your monthly property tax escrow payment.

You have gone over your family's finances and can afford a maximum of \$1,100 per month in total house payments. The total monthly payment includes principal and interest on your loan, mortgage insurance, and property tax escrow.

You go to GoodCo Credit Union and get the following list of their current mortgage programs.

**MORTGAGE RATES FIXED RATE PRIMARY RESIDENCE**  
February 11, 1999

| <b>Term/Type</b> | <b>APR</b> | <b>Rate</b> | <b>Points</b> | <b>Max LTV</b> | <b>Max loan amount</b> |
|------------------|------------|-------------|---------------|----------------|------------------------|
| 30 yr fixed      | 6.822 %    | 6.625 %     | 2.00 %        | 95 %           | \$227,150              |
| 30 yr fixed      | 6.974 %    | 6.875 %     | 1.00 %        | 95 %           | \$227,150              |
| 30 yr fixed      | 7.175 %    | 7.175 %     | 0.00 %        | 95 %           | \$227,150              |
| <b>Term/Type</b> | <b>APR</b> | <b>Rate</b> | <b>Points</b> | <b>Max LTV</b> | <b>Max loan amount</b> |
| 20 yr fixed      | 6.750 %    | 6.375 %     | 2.00 %        | 95 %           | \$227,150              |
| 20 yr fixed      | 6.753 %    | 6.625 %     | 1.00 %        | 95 %           | \$227,150              |
| 20 yr fixed      | 7.000 %    | 7.000 %     | 0.00 %        | 95 %           | \$227,150              |

The six options you should consider are 30 and 20 year loans, at 2, 1, and 0 points. Your assignment is first to determine which of the six home loan programs you can afford, and then choose which of those is the least expensive. For some loans, you may not be able to afford both the points and the minimum down payment. For others, the total monthly payment may be more than you can afford. For each of the loans which you can afford, determine the total cost of the loan and then choose the least expensive. The total cost of the loan is the total interest on the loan plus the points.

Once you choose the best mortgage program, you can use the APR to estimate any prepaid finance charges. (Be sure you understand when to use the APR and when to use the rate of the loan program.) Based on your calculations, determine if GoodCo Credit Union charges any fees besides the points. For the program you have found to be the least expensive, prepare an amortization schedule for the first and last twenty-four months of the loan.

### SOME HINTS FOR THE PROJECT

1. The points are a percentage of the loan amount that you have to pay the credit union at the time you get your loan. For example, if the credit union charges 2.00 percentage points on a loan, and you borrow \$2,000, then you have to pay the credit union  $.02 \times \$2,000 = \$40$  when you get your money. This is *in addition* to the usual interest you have to pay.
2. All the loan programs in the mortgage chart are simple interest amortized loans.
3. One column in the mortgage chart is headed Maximum LTV (Loan to Value). All this means is that the credit union won't lend you more than 95% of what the house costs, so in all cases you must make a down payment of at least 5%.
4. Remember that the more money you put down, the less money you have to borrow. The less money you borrow, the less interest you have to pay. Thus, in each of the six options, you want to make as large of a down payment as possible. However, you only have a fixed amount saved to cover both the down payment and the points. The points are computed on the amount you borrow, namely  $C - D$ , where  $C$  is the sale price of the house and  $D$  is the amount of the down payment. This means that you can find the down payment amount by solving for  $D$  in the equation  $A = p(C - D) + D$ . Here,  $A$  is the amount of money you have for points and down payment and  $p$  is the points written in decimal form (for example, for 3 points,  $p$  would be 0.03). Use this equation with the different point amounts to find the largest possible down payment in each case. Remember though that the down payment must always be at least 5% of the cost of the house.
5. To estimate the taxable value of your property, multiply the price of your house by the percent of fair market value on houses in the town. The tax rate is usually given

as a dollar amount, for example \$1.25, which means that you pay \$1.25 for every \$100 of property value you own. Thus, you can think of the tax rate as a percent—1.25%. Then, to estimate your yearly taxes, multiply the taxable value of your property by the tax rate as a percent. Finally, divide the result by 12 to find the amount of your monthly escrow tax payments.

## STUDENT WORKSHEET: ORGANIZE THE INFORMATION

Use a copy of the following table to list your results for each loan program. If any loan program is unusable, give a reason. For example, you may not be able to afford both the down payment and the points, or the total monthly payments. Once you determine a program is too expensive, you need not finish the computations for that program. Remember that the total monthly payment is the monthly principal and interest payment plus the monthly tax escrow plus the mortgage insurance payment. Also, the total cost of the loan means the total interest plus the cost of any points.

|  |  |
|--|--|
| Number of years                        |  |
| Points                                 |  |
| Rate                                   |  |
| APR                                    |  |
| Number of months                       |  |
| Down payment                           |  |
| Amount of loan                         |  |
| Cost of points                         |  |
| Monthly principal and interest payment |  |
| Total monthly payment                  |  |
| Total interest                         |  |
| Total cost of loan                     |  |

After deciding which program is both affordable and least expensive, use the table below to organize the information for that program.

|   |  |
|---|--|
| Cost of house   |  |
| Total cash on hand  |  |
| Amount needed for closing costs other than points               |  |
| Amount left for points and down payment                         |  |
| 5% minimum down payment   |  |
| Taxable value of your property                                  |  |
| Yearly property taxes   |  |
| Monthly escrow tax payments                                     |  |
| Which program you decided was the best (number of years/points) |  |
| Rate for your program   |  |
| APR for your program  |  |
| Estimated prepaid finance charges for your program              |  |
| How much you paid in points for your program                    |  |

Also, determine if GoodCo Credit Union includes any fees besides the points in your chosen loan program. Then, produce an amortization schedule for the first and last twenty-four months of your chosen program.

## HOUSE BUYING PROJECT SOLUTIONS

### Initial Cash Assessment

|                      |              |   |            |
|----------------------|--------------|---|------------|
| total cash available | \$12,000.00  | maximum total monthly payments          | \$1,100.00 |
| cost of house        | \$129,500.00 | amount left for points and down payment | \$8,500.00 |
| 5% min down paymt    | \$6,475.00   | non-prepaid closing costs               | \$3,500.00 |

### Taxes and Mortgage Insurance

|                    |         |                    |            |
|--------------------|---------|--------------------|------------|
| tax rate           | 0.0193  | fair market value  | 0.86       |
| mortgage insurance | \$75.00 | yearly tax         | \$2,149.44 |
|                    |         | monthly tax escrow | \$179.12   |

### Computational Procedures for the Loan Programs Below

|                      |   |
|----------------------|---|
| number of months     | 12 times the number of years                                    |
| down payment         | computed by solving for $D$ in the formula $A = p(C - D) + D$   |
| amount of loan       | sale price of house minus the down payment                      |
| cost of points       | points (written as a decimal) times the amount of loan          |
| monthly P&I payments | computed using the simple interest amortized loan formula       |
| total monthly pymts  | monthly P&I payments plus monthly taxes plus mortgage insurance |
| total interest       | number of months times monthly P&I payments, minus loan amount  |
| total cost of loan   | total interest plus cost of any points                          |

### Values for Each Loan Program

| 30 years, 2 points  |              | 30 years, 1 point   |              | 30 years, 0 points  |              |
|---------------------|--------------|---------------------|--------------|---------------------|--------------|
| number of years     | 30           | number of years     | 30           | Number of years     | 30           |
| points              | 0.02         | points              | 0.01         | Points              | 0            |
| rate                | 6.625%       | rate                | 6.875%       | Rate                | 7.175%       |
| APR                 | 6.822%       | APR                 | 6.974%       | APR                 | 7.175%       |
| number of months    | 360          | number of months    | 360          | Number of months    | 360          |
| down payment        | \$6,030.61   | down payment        | \$7,277.78   | Down payment        | \$8,500.00   |
| amount of loan      | \$123,469.39 | amount of loan      | \$122,222.22 | Amount of loan      | \$121,000.00 |
| cost of points      | \$2,469.39   | cost of points      | \$1,222.22   | cost of points      | \$0.00       |
| monthly payments    | \$790.59     | monthly payments    | \$802.91     | Monthly payments    | \$819.29     |
| total monthly pymts | \$1,044.71   | total monthly pymts | \$1,057.03   | total monthly pymts | \$1,073.41   |
| total interest      | \$161,142.30 | total interest      | \$166,826.46 | total interest      | \$173,943.30 |
| total cost of loan  | \$163,611.69 | total cost of loan  | \$168,048.68 | total cost of loan  | \$173,943.30 |

| 20 years, 2 points  |              | 20 years, 1 point   |              | 20 years, 0 points  |              |
|---------------------|--------------|---------------------|--------------|---------------------|--------------|
| number of years     | 20           | number of years     | 20           | number of years     | 20           |
| points              | 0.02         | points              | 0.01         | Points              | 0            |
| rate                | 6.375%       | rate                | 6.625%       | Rate                | 7.000%       |
| APR                 | 6.750%       | APR                 | 6.753%       | APR                 | 7.000%       |
| number of months    | 240          | number of months    | 240          | number of months    | 240          |
| down payment        | \$6,030.61   | down payment        | \$7,277.78   | down payment        | \$8,500.00   |
| amount of loan      | \$123,469.39 | amount of loan      | \$122,222.22 | amount of loan      | \$121,000.00 |
| cost of points      | \$2,469.39   | cost of points      | \$1,222.22   | cost of points      | \$0.00       |
| monthly payments    | \$911.49     | monthly payments    | \$920.27     | monthly payments    | \$938.11     |
| total monthly pymts | \$1,165.61   | total monthly pymts | \$1,174.39   | total monthly pymts | \$1,192.23   |
| total interest      | \$95,288.43  | total interest      | \$98,643.24  | total interest      | \$104,146.81 |
| total cost of loan  | \$97,757.82  | total cost of loan  | \$99,865.46  | total cost of loan  | \$104,146.81 |

## ANALYSIS

Note that paying two points does not leave enough money for the minimum down payment of \$6,475. Furthermore, the total monthly payments on all of the 20 year loans is higher than \$1,100.

Thus, the only possible loans are the 30 year mortgages with either 1 or 0 points. Of the two, the 1 point program has the lower total cost, and hence is the best long term mortgage.

The best program which is affordable is a 30 year loan with one point.

### Estimating Prepaid Finance

#### Charges

estimated prepaid finance charges \$1,220.91

difference between estimated charges and cost of points -\$1.31

Thus, there are no prepaid finance charges other than the point.

### Amortization Schedule for Selected Months

| Month Number | Principal Portion | Interest | Monthly Payment | Amount Due After Payment |
|--------------|-------------------|----------|-----------------|--------------------------|
| 0            | ---               | ---      | ---             | \$122,222.22             |
| 1            | \$102.68          | \$700.23 | \$802.91        | \$122,119.54             |
| :            | :                 | :        | :               | :                        |
| 24           | \$117.10          | \$685.81 | \$802.91        | \$119,588.47             |
| :            | :                 | :        | :               | :                        |
| 337          | \$700.04          | \$102.87 | \$802.91        | \$17,255.82              |
| :            | :                 | :        | :               | :                        |
| 360          | \$798.34          | \$4.57   | \$802.91        | \$0.00                   |