

# CREDIT CARDS AND CARS:

## THE MATHEMATICS OF THE AMERICAN DREAM PART I

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**Description:** Given an actual credit card statement, students verify the bill. With materials from a local automobile dealership, students choose between financing and leasing a new car.

**Keywords:** Credit cards, car payments, simple interest, finance charges, down payments, simple interest amortized loans, financial mathematics.

**Objectives:** To engage students with hands-on experience with some of the calculations and analysis involved in balancing a credit card statement and choosing whether to buy or lease a car; to provide motivation for various topics in financial mathematics; to convince students of the relevance of mathematics in their daily lives.

**Prerequisites:** Organizational ability, attention to detail, and comfort with algebraic concepts such as order of operation and properties of exponents.

**Course level:** Any course, such as algebra II, pre-calculus, financial mathematics, in which students have or develop an understanding of basic algebraic methods, or any course that develops basic formulas for simple and compound interest, and amortized loans. This is a good capstone project for the financial component of such a course.

**Technology used:** At least a calculator with a memory function and exponentiation key.

**Required equipment/materials:** To modify the project for local conditions, a current credit card bill, as well as a loan and a lease form from a local automobile dealership.

**Time estimates:** *Instructor preparation:* Minimal for project provided here, 2 to 6 hours for a customized project using local information. *Class presentation:* Minimal in a class that already includes basic finance formulas, several class periods to develop the formulas as ancillary material. *Student work outside of class:* About 4 to 8 hours. *Fielding questions:* Average of about 30 minutes per group. *Grading:* Roughly 20 minutes per project submitted.

**NCTM standards addressed:** *Content standards:* number and operations, algebra. *Process standards:* problem solving, communication, connections, representation.

Credit cards and cars. What could be a bigger part of the consumer consciousness? And what better means of impressing students with the immediate relevancy of mathematics in their lives? This is especially true since credit cards and car loans are often the first major financial commitments students make. But to be most convincing, the context of the financial mathematics should be real — an actual credit card bill or pages of fine print from a car dealer — not just another formula-tailored exercise in a textbook.

With this in mind, we started searching for classroom materials (included below) for our students. We found a credit card bill with sufficient complexity to expose students to some of the challenges of reading their monthly statements. We visited the local Saturn dealership and met with a very helpful finance officer, who gave us glossy picture brochures and mocked up the paperwork for buying or leasing a shiny new car. Then, we brought this material to our students and watched them delve into it. Since many of our students have credit cards, and have (or want!) cars, this material immediately attracted and held their interest. They exerted themselves considerably beyond what they would have for textbook exercises, and as a result learned and retained far more, with far greater enthusiasm. Students have commented on how valuable just the exposure to the technical details of the actual paperwork was.

The documents we collected eventually evolved into the following two projects: first, reading a credit card bill, and second, determining whether to lease or borrow money for a new car. We have used these two projects successfully for many years and

updated them regularly. They require only very elementary financial mathematics, such as simple interest and amortized loan formulas. Because of this, they fit well into many different courses, including high school algebra, pre-calculus, finite mathematics, mathematics education, college algebra, and elementary business. They can be used to motivate mathematics ranging from the basic algebra skills of simplification, order of operations, and exponentiation, up through laying the foundation for exponential functions (compound interest), and evaluating finite sums (amortization formulas).

Modifying these projects to add local flavor or for repeated use is very easy. After removing identifying information, any non-trivial credit card bill will work. A car dealership will have loan and lease forms. We have had good experiences at the local Saturn dealership, but the average salesperson working on commission may not be as accommodating. It helps to call ahead to set up an appointment and explain the project. We use a spreadsheet program to generate solutions after updating a project. Since the formulas remain essentially the same, entering new raw data in the appropriate cells then generates answers for the whole project.

In the credit card project, students determine simple interest, the monthly periodic rate, the average daily balance, and the finance charge. For the statement given below, the finance charge is the average daily balance times the monthly periodic rate. Another way credit card companies sometimes compute the finance charge is with  $F.C. = Prt$ , where  $P$  is the principal or average daily balance,  $r$  is the annual interest rate, and  $t$  is the time in years. If the time  $t$  is the number of days out of 365 instead of one month out of twelve, this equation will give a slightly different value. Students should also know the difference between posted and transaction dates and between debits and credits.

In the car project, students work through the finances of two options. The first is borrowing money to buy a new car, and the second is leasing the car for three years, then exercising the option to buy. To find the amount of the monthly car payments, they will need to use the following amortized loan equation:  $P(1+i)^n = \text{payment} \frac{(1+i)^n - 1}{i}$ , where  $P$  is the principal,  $i$  is the periodic rate, and  $n$  is the number of payments. They will also need to find the total interest using:  $\text{Total interest} = n * \text{payment} - \text{amount financed}$ .

Some other charges involved in the car project include license and registration fees, sales or use taxes, and finance charges for a car loan. Students will not have to compute the fees and taxes, but they should be aware that these add significantly to the sticker price of the car. This is often a big shock to students who have not experienced a car sale before!

Because this project includes actual loan and lease forms from a car dealership, there is a lot of extraneous information. In fact, due to space considerations, two pages of legalese were omitted for this article. While the forms may be a little confusing initially, the experience of working through genuine paperwork proves very valuable to most students.

The car project is constructed so that there is very little difference between the total cost of the two options. Thus, students must consider different lifestyles in evaluating whether to buy or lease. Students may make a case for options other than the two presented in the project. For example, after leasing the car for three years, they may want to lease another car instead of exercising their option to buy. Some students may

make a case for saving more money and making a substantial down payment or even paying cash for the car.

Because of this, evaluating the third part of the project requires some flexibility. It is important to emphasize to students that they must support their decision with financial analysis. For example, students who advocate paying cash for the car should at least calculate the amount of money saved. Hopefully, they will also address the question of what to do for transportation while saving for the new car!

The classroom materials and solutions for both projects are below.

## CREDIT CARD PROJECT

Organize the information on the credit card statement below in a daily balance table like the following. Then, determine the amounts that go in boxes 1, 2, and 3 of the statement below.

Time Interval	# of Days	Transaction Total	Daily Balance
---	---	---	(beginning balance)

This statement covers the period from 3/8/98 to 4/7/98, inclusive. When filling out the table, use the date of transaction (abbreviated **trans** on the bill) or the initial date of the billing period (3/8/98), whichever is later. The posted date is just when the credit card company received notice of the transaction, and is not used to compute any part of the bill.

The time interval spans the period from the day of the transaction to the day before the next transaction, inclusive. To find the transaction total, sum all the transactions that occur during the time interval. Then, add (or subtract in the case of a credit) this total to the daily balance. Any transactions which occur before the start of the billing period must be included in the daily balance for the first time interval. Some transaction totals may include more than one transaction occurring on the same day.

address \_\_\_\_\_  
 city state zipcode \_\_\_\_\_  
 area code telephone number \_\_\_\_\_

Account Number	Payment Due Date	New Balance	Minimum Payment Due
	05/01/98	4,010.46	52.00
Please write amount paid			
\$			

If address shown is incorrect, please make correction.

Do Not Send Cash

ACCOUNT TYPE		ACCOUNT NUMBER	CREDIT LINE	AVAILABLE CREDIT	BILLING DATE	PAYMENT DUE DATE
PREFERRED MASTERCARD					04/07/98	05/01/98
DATE		REFERENCE NUMBER	TRANSACTION DESCRIPTION			DEBITS/CREDITS
POSTED	TRANS					
03/10	03/06	XXXXXXXXXX00000000	CARRBORO FAMILY MEDICI	CARRBORO	NC	239.00
03/17	03/13	XXXXXXXXXX00000000	PIECE GOODS SHOP #8	CHAPEL HILL	NC	250.71
03/25	03/20	XXXXXXXXXX00000000	U.S. AIR	CHAPEL HILL	NC	608.00
03/30	03/26	XXXXXXXXXX00000000	DILLARDS-UNIVERSITY	CHAPEL HILL	NC	38.41
03/30	03/26	XXXXXXXXXX00000000	NOWELL'S CHP HILL INC	CHAPEL HILL	NC	231.76
03/30	03/28	XXXXXXXXXX00000000	ROSCOE GRIFFINS 5005	DURHAM	NC	53.14
03/30	03/26	XXXXXXXXXX00000000	NOWELL'S CHP HILL INC.	CHAPEL HILL	NC	82.68
03/31	03/29	XXXXXXXXXX00000000	DURHAM BULLS SUBSIDIAR	RALEIGH	NC	210.55
03/31	03/28	XXXXXXXXXX00000000	BELK 20	DURHAM	NC	16.96
03/31	03/28	XXXXXXXXXX00000000	T J MAXX 320	DURHAM	NC	328.58
04/03	04/03	XXXXXXXXXX00000000	PAYMENT - THANK YOU			622.41-

**START YOUR SPRING OFF RIGHT! TAKE ADVANTAGE OF THE ENCLOSED COUPONS FOR GREAT SAVINGS EVERY TIME YOU RENT FROM HERTZ!**

SUMMARY	PREVIOUS BALANCE	PURCHASES ADVANCES AND DEBITS	PAYMENTS AND OTHER CREDITS	FINANCE CHARGE		NEW BALANCE	AMOUNT PAST DUE INCLUDED IN MINIMUM PAYMENT	YOUR MINIMUM PAYMENT
				At periodic rate	Transaction Fees			
PURCHASES ADVANCES	2,522.36 .00	2,059.79 .00	622.41 .00	<b>3</b>	.00	4,010.46 .00		
TOTALS	2,522.36	2,059.79	622.41		.00	4,010.46		52.00

YOUR FEE REBATE STATUS: USAGE TO DATE = \$1,205. ADDITIONAL USAGE NEEDED BY 01/01 TO REACH \$12.50 REBATE = \$3,795. ADDITIONAL USAGE NEEDED BY 01/01 TO REACH \$25 REBATE = \$6,295.

RATES APPLIED TO BALANCES				
TRANSACTIONS	BALANCE SUBJECT TO FINANCE CHARGE	MONTHLY PERIODIC RATE	NOMINAL A.P.R.	ANNUAL PERCENTAGE RATE
PURCHASES	<b>2</b>	<b>1</b>	16.900%	16.900%
ADVANCES			16.900%	16.900%

\* YOU CAN AVOID ANY FINANCE CHARGES ON \*  
 \* PURCHASES IF YOU PAY YOUR PURCHASES \*  
 \* BALANCE IN FULL EVERY MONTH. \*

**Figure 1. Credit Card Statement.**

## CREDIT CARD SOLUTIONS

### Daily Balance Table

Time Interval	# of Days	Transaction Total	Daily Balance
			\$2,522.36 (beginning balance)
March 8-12	5	\$239.00	\$2,761.36
March 13-19	7	\$250.71	\$3,012.07
March 20-25	6	\$608.00	\$3,620.07
March 26-27	2	\$352.85	\$3,972.92
March 28-28	1	\$398.68	\$4,371.60
March 29-April 2	5	\$210.55	\$4,582.15
April 3-April 7	5	-\$622.41	\$3,959.74

The transaction total is the sum total of all transactions occurring in the given time interval.

Since the first transaction of \$239 occurred on March 6 before the start of the billing period on March 8, this amount is added to the daily balance for March 8-12.

Also, the daily balances for the intervals March 26-27 and March 28-28 each include three charges:

for March 26-27,  $\$352.85 = \$38.41 + \$231.76 + \$82.68$

and for March 28-28,  $\$398.68 = \$53.14 + \$16.96 + \$328.58$ .

### Monthly Periodic Rate (Box 1)

The rate is found by dividing the Annual Percentage Rate by 12:  $16.9\% / 12 = 1.41\%$ .

### Average Daily Balance (Box 2)

The A.D.B. is a weighted average of the daily balances during the credit card period, and is found by multiplying the number of days in each interval by the balance during that interval, then summing and dividing by the number of days in the month:

$$\{5(\$2,761.36) + 7(\$3,012.07) + 6(\$3,620.07) + 2(\$3,972.92) + 1(\$4,371.60) + 5(\$4,582.15) + 5(\$3,959.74)\} / 31 \\ = \$111,638.60 / 31 = \$3,601.25.$$

### Finance Charge (Box 3)

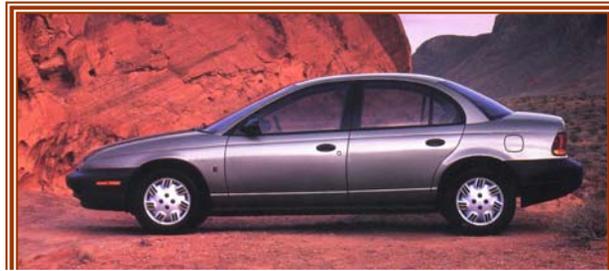
This charge is found using the simple interest formula over the one month period of the bill.

It can be found by multiplying the A.D.B. by the periodic rate as follows:

$$\text{Finance charge} = \text{A.D.B} \times \text{monthly periodic rate} = \$3,601.25(0.169)(1/12) = \$50.72.$$

## CAR LOAN/LEASE PROJECT

You have decided that you want the new Saturn pictured below. The sticker price is \$13,170.00. You could either get a loan for the car, or you could lease it for a while and then buy it. The goal of this project is for you to determine which is the best plan. This project includes the paperwork from a Saturn dealership for two options. The first is to buy the car by making an \$800.00 down payment and financing the remainder with a five-year simple interest amortized loan. The second is to lease the car for three years, and then, at the end of the three years, take out a two-year simple interest amortized loan for the remaining value of the car.



**Figure 2. Saturn automobile.**

The first form below is a copy of an actual car loan for a new Saturn. You must fill in boxes A through F (box D is just the amount of the down payment). This is a simple interest amortized loan at 9.9%, and all the information you need is in the document if you read carefully.

The second attachment below is a copy of a lease package for the same new Saturn. Suppose you lease the car for the three years offered in the package, then exercise your option to buy the car at the end. You get a simple interest amortized loan at 9.9% for two years with monthly payments. Show your computations for what your monthly payments would be. Also, compute the total amount you would be spending on the car including both the loan payments and the three years of lease payments.

Then, discuss the advantages and disadvantages of buying the car. Discuss the advantages and disadvantages of leasing the car. Make a case for doing one or the other or propose another course of action. In making your case, you do not have to assume you will exercise your option to buy the leased car. You must support your argument with financial figures and life style scenarios that validate your argument.



**CLOSED-END VEHICLE LEASE AGREEMENT**  
 NOTE: A TABLE OF CONTENTS FOR THIS LEASE APPEARS ON THE REVERSE SIDE

Saturn of South Burlington

1089 Shelburne Road, South Burlington, VT 05403

Lessor—Dealer Name  
**Mickey M. Mouse**

Address  
 123 Disney Drive, Orlando, FL 05404

Lessee—Name(s)

Address(es)

**1. IDENTIFICATION OF PARTIES AND VEHICLE:** You (meaning each lessee signing this lease) agree to lease from us (meaning the lessor) the vehicle described below. The vehicle may be subject to certain express warranties. Unless the box at the end of this Item is checked, the vehicle is subject to a manufacturer's new car warranty. The vehicle is also subject to any other express warranties or guarantees identified at the end of this Item 1. We are assigning to you all our rights and remedies under any such express warranties or guarantees. The lease term and obligations begin when the lease is signed by you and by us. This lease includes consumer lease disclosures, which are also made on behalf of Chase Manhattan Automotive Finance Corporation, to whom the lessor intends to assign this lease, unless a different assignee is indicated:

YEAR	MAKE	MODEL	BODY STYLE	COLOR	IDENTIFICATION NO.	LICENSE (Where Known)	IGNITION KEY
1998	SATURN	SL1	4D SEDAN	DARK			0295

<input checked="" type="checkbox"/> Air Conditioning	<input type="checkbox"/> Power Seats	<input checked="" type="checkbox"/> Power Steering	Engine (Type) <u>4 Cylinder</u>
<input type="checkbox"/> Power Windows	<input checked="" type="checkbox"/> Power Brakes	<input checked="" type="checkbox"/> Tilt Wheel	Trans (Type) <u>5 Speed</u>
<input type="checkbox"/> Cruise Control	<input type="checkbox"/> Power Door Locks	<input type="checkbox"/> Other _____	Radio (Type) <u>Cassette</u>

If this box is checked, the vehicle is not subject to a manufacturer's new car warranty. If the vehicle is subject to another express warranty or guarantee, such other express warranty or guarantee is disclosed here: \_\_\_\_\_

**2. VEHICLE RECEIPT:** You acknowledge that: (a) you have received and examined the vehicle described above; (b) it is equipped as described and in good operating order and condition; (c) you accept it for all purposes of this lease; and, (d) the odometer reading is \_\_\_\_\_ miles.

**3. AMOUNTS YOU WILL PAY WHEN SIGNING THIS LEASE:**

a. Credit to initial Value of Vehicle	
1. Cash Payment	\$ <u>N/A</u>
2. Trade-in:	
Year _____ Make _____	\$ <u>N/A</u>
3. Sales or Use Tax (on a1 and/or a2)	\$ <u>N/A</u>
4. Total (a1+a2+a3)	\$ <u>N/A</u>
b. Security Deposit	\$ <u>N/A</u>
c. First Monthly Payment (same as Item 4e)	\$ <u>233.40</u>
d. First Year's License and Reg. Fee	\$ <u>53.00</u>
e. First Year's Personal Property Tax	\$ <u>N/A</u>
f. Life and/or Disability Insurance	\$ <u>N/A</u>
g. Mechanical Breakdown Protection	\$ <u>N/A</u>
h. Other: <u>Doc Fees</u>	\$ <u>68.00</u>
i. Sales or Use Tax (on g and/or h)	\$ <u>316.08</u>
j. Other Sales or Use Tax	\$ <u>N/A</u>
k. Total (sum of a4 through j)	\$ <u>670.48</u>

**4. TERM, MONTHLY PAYMENTS AND LATE CHARGES:** The 1<sup>st</sup> monthly payment is due when you sign this lease. You will make a monthly payment each subsequent month during the lease term as follows:

a. Day of Month Each Payment is Due	<u>4th</u>
b. Lease Term (number of months)	<u>36</u>
c. Monthly Rent Payment	\$ <u>233.40</u>
d. Sales or Use Tax (c x tax rate)	<u>N/A</u>
e. Total Monthly Payment (c+d)	\$ <u>233.40</u>
f. Total of All Monthly Payments (b x e)	\$ <u>8,402.40</u>

(\*Estimate based on current sales or use tax rate)

**5. ESTIMATED FEES AND TAXES:** You will pay when due or reimburse us for (or advance at least termination) all governmental fees and taxes, including personal property taxes, if applicable, other than our net income taxes, concerning the vehicle and this lease. We estimate the total amount to be 316.08;

**6. REQUIRED INSURANCE:** You must maintain in full force and effect the following insurance until you return the vehicle to us: (a) **\$100,000/\$300,000** for bodily injury and **\$50,000** for property damage listing us as Additional Insured; and (b) collision, fire, theft and comprehensive coverage in an amount equal to the fair market value of the vehicle with a maximum deductible of **\$500** listing us as Loss Payee and Additional Insured. The insurance policy you have must be approved by us and must state that we will be given at least **10** days notice of any cancellation, reduction or other material change in coverage. You will furnish whatever written proof of the required coverage we may request. Any dishonor by the insurer of your insurance policy or any claim made under your policy due to your misrepresentations or other acts or omissions, will mean that you have not maintained the insurance required by this paragraph, and you will be responsible for any loss that may result from the insurer's dishonor of your policy or claim.

**9. USE OF VEHICLE:** You intend to use the vehicle (initial only applicable box):

CONSUMER LEASE – Primarily for personal, family or household purposes; or

COMMERCIAL LEASE - Primarily for business, commercial or agricultural purposes, or you are an organization or governmental entity.

Lessee(s) Initials

Lessee(s) Initials

**10. PURCHASE OPTION:** You acknowledge that this is a true lease and you will not own or have any equity in the vehicle or its replacement parts unless you exercise the purchase option, if any.

a. You have the option to purchase the vehicle from us on an AS-IS WHERE-IS basis at early or scheduled termination. The purchase price will be the sum of:

- i. A fee of \$ 150.00 to purchase the vehicle (if no amount is indicated, a fee of \$150 applies);
- ii. All amounts you owe under this lease that are not included in Items 10a iii thru 10a v;
- iii. Any official fees and taxes imposed in connection with the purchase of the vehicle; plus either;
- iv. If the purchase is at early termination, the greater of the adjusted lease balance as calculated in Item 11d or the fair market value as agreed to by the parties. If you and we cannot agree upon a fair market value within **10** days, you have the right to obtain, at your expense, a professional appraisal of the fair market value, if we can both agree to your selection of a qualified independent appraiser; or
- v. If the purchase is at scheduled termination, the fair market value of the vehicle at scheduled termination, which you and we have discussed and determined to be:

\$ 7,902.00  
 (if option applies, cross out Item 10b); or

Lessee(s) Initials

b. You have no option to purchase the vehicle and must return the vehicle to the place we specify (if no option, cross out Item 10a).

Lessee(s) Initials

**11. EARLY TERMINATION:** You may end this lease on the due date of a monthly payment, if you are not in default. You must give us 30 days' written notice that you intend to end this lease. Unless you buy the vehicle, you must return to a place we specify. At any time during this lease, we may end the lease if you default. For a list of default events, see Item 18. We may also end this lease if the perils described in Item 17 occur. If you do not buy the vehicle, unless the last sentence of Item 6 applies, you will owe us at early termination the sum of:

- a. A fee of \$ N/A to dispose of the vehicle (if no amount is entered a fee of \$300 applies);
- b. All amounts you owe under this lease that are not included in Items 11c and d;
- c. Any official fees and taxes imposed in connection with lease termination; and
- d. Any amount by which the then adjusted lease balance exceeds the vehicle's realized value at early termination. To compute the adjusted lease balance, we multiply the average monthly depreciation of: \$ 160.08 by the total number of monthly payments (4b)

**Figure 4. Saturn Automobile Lease.**

## CAR LOAN/LEASE SOLUTIONS

### 5 Year Loan Option

interest rate	9.90%			
number of months	60			
cash price of car	\$13,170.00	<b>Box</b>	<b>Answer</b>	<b>How Computed</b>
fees:				
documentation	\$68.00	<b>A</b> (finance charge)	\$3,611.60	total monthly pymts - amount financed
registration	\$43.00			
sales tax	\$790.20	<b>B</b> (amount financed)	\$13,286.20	
title	\$15.00			
total fees	\$916.20	<b>C</b> (total pymts)	\$16,897.80	monthly payment x number of months
cost to buy car	\$14,086.20			
down pymt	\$800.00	<b>D</b> (down pymt)	\$800.00	
amount of loan	\$13,286.20	<b>E</b> (total sale price)	\$17,697.80	all monthly pymts + initial down pymt
		<b>F</b> (amount of pymts)	\$281.63	simple interest amortized loan formula--answer rounded down

### 3 Year Lease Then Purchase Option

#### The 3 Year Lease

monthly payments	\$233.40
Number of months	36
total paid on lease	\$8,402.40

fees:

documentation	\$68.00
first year lic. and reg.	\$53.00
sales tax	\$316.08
total fees	\$437.08

total cost for first three years	\$8,839.48
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#### The 2 Year Purchase Option

termination value of car	\$7,902.00	interest rate	9.90%
fee	\$150.00	number of months	24
amount of loan	\$7,902.00		

<b>Value</b>	<b>Answer</b>	<b>How Computed</b>
finance charge	\$840.48	total monthly pymts - amount financed
amount financed	\$7,902.00	
total payments	\$8,742.48	monthly payment x number of months
amount of payments	364.27	simple interest amortized loan formula--answer rounded down

#### Total Cost for the Lease/Purchase Option

total for first 3 years + total loan pymts + \$150 fee	\$17,731.96
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### Comparing the Two Options

total for loan option	\$17,697.80	total for lease/buy option	\$17,731.96	difference in options	-\$34.16
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