Astronomy (PY 101)

NAME ________________________

Project VII: White Dwarfs

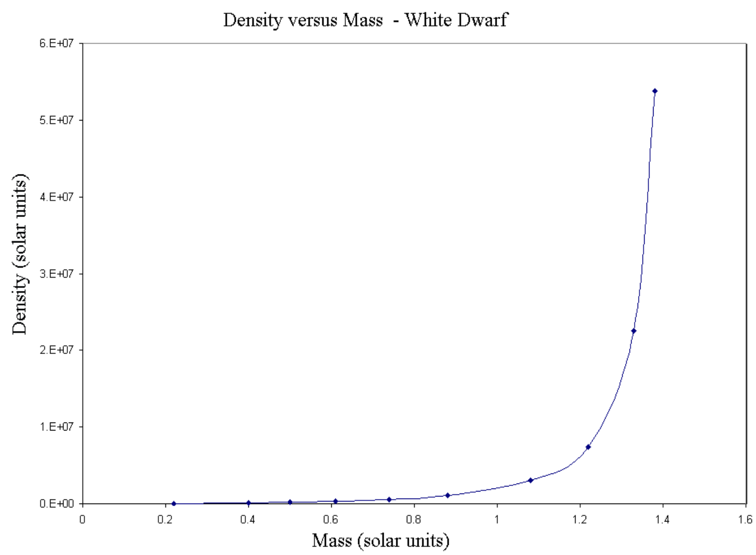
The white-dwarf (WD) phase is the last evolutionary step in the life of a non-massive star ($\mathcal{M} < 6\ \mathcal{M}_\odot$). White-dwarf stars are objects of tremendously large densities.

It turns out that the mass of a white-dwarf (WD) star cannot exceed a theoretical maximum value of 1.44 $\mathcal{M}_\odot$. As the WD star approaches this maximum value, its theoretical radius R reaches zero, i.e., its density approaches infinity! This incredible result simply means that the state of matter known as a white dwarf can no longer exist as such and is replaced by a different state (more exotic) state of matter. In practice, the density of a white dwarf star cannot exceed a critical value $\rho_c = 2.40 \times 10^{11}$ kg/m^3 (240 million times the density of water).

The Table below contains the theoretical WD radius R (expressed in solar radius units $R_\odot = 6.96 \times 10^8$ m) for several values of WD mass $\mathcal{M}$ (expressed in solar mass units $\mathcal{M}_\odot = 1.99 \times 10^{30}$ kg). For comparison, the density $\rho = \mathcal{M}/(\frac{4\pi}{3}R^3)$ of each white dwarf star listed here is also given (expressed in critical density units ρ_c); note that the density of the Sun is $\rho_\odot = 1.41 \times 10^3$ kg/m^3 (1.41 times the density of water).

Mass Ratio $\mathcal{M}/\mathcal{M}_\odot$	Radius Ratio $R/R_\odot$	Density Ratio $\rho/\rho_\odot$	Density Ratio ρ/ρ_c
0.22	19.95×10^{-3}	0.028×10^6	1.63×10^{-4}
0.40	15.49×10^{-3}	0.108×10^6	6.32×10^{-4}
0.50	13.80×10^{-3}	0.190×10^6	11.1×10^{-4}
0.61	12.30×10^{-3}	0.328×10^6	19.2×10^{-4}
0.74	10.96×10^{-3}	0.562×10^6	33.0×10^{-4}
0.88	9.33×10^{-3}	1.083×10^6	63.6×10^{-4}
1.08	7.08×10^{-3}	3.043×10^6	0.018
1.22	5.49×10^{-3}	7.373×10^6	0.043
1.33	3.89×10^{-3}	22.6×10^6	0.13
1.38	2.95×10^{-3}	53.7×10^6	0.32

The Figure below shows the density of a white-dwarf star (in solar units) as a function of its mass (in solar units). Notice how the density increases rapidly as the mass of the white dwarf approaches the theoretical limit of 1.44 $\mathcal{M}_\odot$.



Part I. Using the graph paper provided, plot $R/R_\odot$ (vertical axis) versus $\mathcal{M}/\mathcal{M}_\odot$ (horizontal axis).

Part II.

Sirius B and Procyon B are two well-known white-dwarf stars. Their observed properties are listed in the Table below.

Star	Spectral Type	Parallax $\Pi('')$	Apparent Magnitude m	Temperature Ratio $T/T_\odot$
Sirius B	O5 ($26,000\ K$)	0.377	5.9	4.50
Procyon B	A8 ($8,700\ K$)	0.285	10.7	1.51

(a) Calculate the distances D of Sirius B and Procyon B.

Star	Parallax $\Pi('')$	Distance D(pc)
Sirius B	0.377	
Procyon B	0.285	

(b) Calculate the absolute magnitudes M of Sirius B and Procyon B.

Star	Distance D(pc)	Apparent Magnitude m	Absolute Magnitude M
Sirius B		5.9	
Procyon B		10.7	

(c) Calculate the luminosity ratios $L/L_\odot$ of Sirius B and Procyon B by using the formula

$$\frac{L}{L_\odot} = 10^{(M_\odot - M)/2.5}$$

where the absolute magnitude of the Sun is $M_\odot = 4.73$.

Star	Absolute Magnitude $M_\odot - M$	Luminosity Ratio $L/L_\odot$
Sirius B		
Procyon B		

(d) Calculate the radius ratios $R/R_\odot$ of Sirius B and Procyon B by using the formula

$$\frac{R}{R_\odot} = \frac{\sqrt{L/L_\odot}}{(T/T_\odot)^2}$$

Star	Temperature Ratio $T/T_\odot$	Luminosity Ratio $L/L_\odot$	Radius Ratio $R/R_\odot$
Sirius B	4.50		
Procyon B	1.51		

(e) From your graph ($R/R_\odot$ versus $\mathcal{M}/\mathcal{M}_\odot$), find the masses of Sirius B and Procyon B.

Star	Radius Ratio $R/R_\odot$	Mass Ratio $\mathcal{M}/\mathcal{M}_\odot$
Sirius B		
Procyon B		