Lab 03 CLEA The Revolution of the Moons of Jupiter

Purpose

Determine the mass of Jupiter by measuring the orbital properties of Jupiter's moons and using Kepler's Third Law.

Historical Background

Astronomers cannot directly measure the masses and distances of the planets and their moons. Nevertheless, we can deduce some properties of celestial bodies from their motions despite the fact that we cannot directly measure them. In 1543 Nicolaus Copernicus hypothesized that the planets revolve in circular orbits around the sun. Tycho Brahe (1546-1601) carefully observed the locations of the planets and 777 stars over a period of 20 years using a sextant and compass. These observations were used by Johannes Kepler, an assistant of Brahe's, to deduce three empirical mathematical laws governing the orbit of one object around another. Kepler's Third Law is the law that applies to this lab. For a moon orbiting a much more massive parent body, the Third Law states the following:

$$\mathbf{M} = \frac{\mathbf{a}^3}{\mathbf{p}^2}$$

where

M is the mass of the parent body in units of the mass of the sun

a is the length of the semi-major axis in units of the mean Earth-Sun distance, 1 A.U. (astronomical unit). If the orbit is circular (as we assume in this lab), the semi-major axis is equal to the radius of the orbit.

p is the period of the orbit in Earth years. The period is the amount of time required for the moon to orbit the parent body once.

In 1609, Galileo used a telescope to discover that Jupiter had four moons orbiting it and made exhaustive studies of this system, which was especially remarkable because the Jupiter system is a miniature version of the solar system. Studying such a system could open a way to understand the motions of the solar system as a whole. Indeed, the Jupiter system provided clear evidence that Copernicus' heliocentric model of the solar system was physically possible.

Introduction

We will observe the four moons of Jupiter that Galileo saw through his telescope, known today as the Galilean moons. They are named Io, Europa, Ganymede and Callisto, in order of distance from Jupiter. You can remember the order by the mnemonic "**I E**at **G**reen **C**arrots." If you looked at Jupiter through a small telescope, you might see the following:

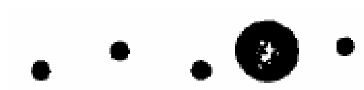

Figure 1
Jupiter and Moons through a Small Telescope

The moons appear to be lined up because we are looking edge-on at the orbital plane of the moons of Jupiter. If we watched, as Galileo did, over a succession of clear nights, we would see the moons shuttle back and forth, more or less in a line. While the moons actually move in roughly circular orbits, you can only see the perpendicular distance of the moon to the line of sight between Jupiter and Earth. If you could view Jupiter from "above" (see Figure 2), you would see the moons traveling in apparent circles.

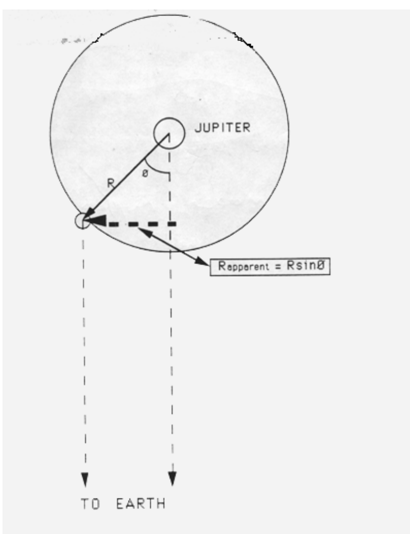

Figure 2

VIEW FROM ABOVE THE PLANE OF ORBIT

$R_{apparent}$ shows the apparent distance between the moon and Jupiter that would be seen from earth.

As you can see from Figure 3, the perpendicular distance of the moon should be a sinusoidal curve when plotted over time. By taking enough measurements of the position of a moon, you can fit a sine curve to the data and determine the radius of the orbit (the amplitude of the sine curve) and the period of the orbit (the period of the sine curve). Once you know the radius and period of the orbit of that moon you can determine the mass of Jupiter using Kepler's Third Law. The mass of Jupiter will be determined using measurements of each of the four moons; there will be errors of measurement associated with each moon, and therefore your Jupiter masses may not be exactly the same.

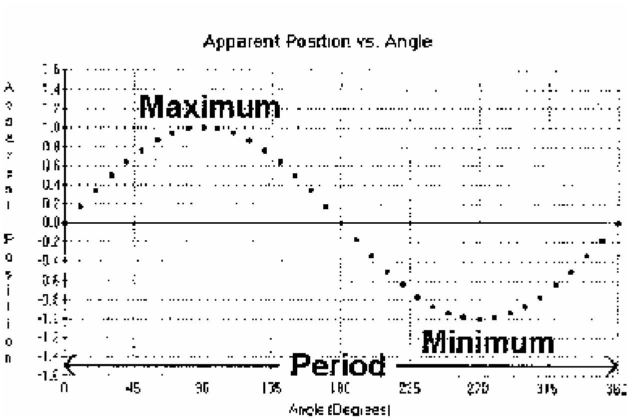

Figure 3

GRAPH OF APPARENT POSITION OF A MOON

The apparent position of a moon varies sinusoidally with the changing angle from the line of sight, θ , as it orbits Jupiter. Here the apparent position is measured in units of the radius of the moon's orbit, **R**, and the angle measured in degrees.

This program simulates the operation of an automatically controlled telescope with a camera that provides a video image to a computer screen. It also allows convenient measurements to be made at a computer console, as well as adjustment of the telescope's magnification. The computer simulation is realistic in all important ways, and using it will give you a good understanding of how astronomers collect data and control their telescopes. Instead of using a telescope and actually observing the moons for many days, the computer simulation shows the moons to you as they would appear if you were to look through a telescope at the specified time.

Overall Strategy:
This is the overall plan of action for this laboratory exercise.

- Start up the program and use it to familiarize yourself with the Jupiter system.
- Set up observing sessions.
- Measure positions of Jupiter's moons over successive clear nights.
- Plot a graph of your observations for each moon, using the ***Revolution of Jupiter's Moons*** program.
- Using this program to help you, fit a sine curve to each graph.
- Determine the period and semi-major axis for the orbit of each moon from its graph and then convert the values to years and AU, respectively.
- Calculate the mass of Jupiter from your observations of each moon, then determine the average value for Jupiter's mass from your individual values.

Before You Start

Now is an ideal time to have a little fun with the program and in the process visualize what you will be doing and why. Start up the **Jupiter's Moons** lab; then select **Log in . . .** from the **File** menu. Enter your full name in the dialogue box that appears and select **OK**. Now select **File . . .** > **Run . . .** ; when the next window pops up, simply select **OK** to accept the defaults for the **Start Date & Time**; you will be going back to change these after you've familiarized yourself with the program and the motions of the Jupiter system. Now the window pictured below appears, showing Jupiter much as it would appear in a telescope. Jupiter appears in the center of the screen, while the small, point-like moons are on either side. Sometimes a moon is hidden behind Jupiter and sometimes it appears in front of the planet and is difficult to see. You can display the screen at four levels of magnification by clicking on the **100X**, **200X**, **300X**, and **400X** buttons. The screen also displays the date, Universal Time (the time at Greenwich, England), the Julian Date (a running count of days used by astronomers; the decimal is an expression of the time), and the interval between observations (or animation step interval if **Animation** is selected).

Figure 4

To do something you can't do with the real sky, select **File** > **Preferences** > **Animation**, then click on the **Cont.** (Continuous) button on the main screen. Watch the moons zip back and forth as the time and date scroll by. With this animation, it's fairly easy to see that what the moons are really doing is circling the planet while you view their orbits edge-on. To reinforce this, stop the motion by selecting **Cont.** again, select **File** > **Preferences** > **Top View**, and start the motion again (**Cont.**). Note that under the **Preferences** menu you can also choose **ID Color** and avoid confusing the four moons. When you are satisfied that you understand the motions of Jupiter's moons and why they appear the way they do, you are ready to start the lab.

Turn off the <u>Animation</u> feature before going on to the next section.

Procedure

Stop the motion of the moons and select **Run . . .** again, the **Start Date & Time** window will appear and now you will change the defaults to today's date and 0 hrs. Each person will be assigned to collect data for one Moon. Each Moon uses a different observation interval.

Name of assigned moon ___________________

The observation intervals are: Io = 6 hrs, Europa = 12 hrs, Ganymede = 24 hrs, Callisto = 36 hrs. Select **File, Preferences, Timing** to change the observation interval.

In order to measure the position of a moon, move the pointer to the moon and left-click the mouse. The lower right-hand corner of the screen will display the name of the moon (for example, **II. Europa**), the X and Y coordinates of its position in pixels on your screen, and its X coordinate expressed in Jupiter diameters to the east or west of the planet's center. This is the crucial figure for our purposes. Note that if the name of the moon does not appear, you may not have clicked exactly on the moon, so try again. To measure each moon's position accurately, switch to the highest magnification that will keep the moon on the screen.

You will also use the computer to record your data for later analysis. After taking a measurement, select **Record Measurements . . .** and enter the data in the dialog box that appears. You can go back and add to or edit this data later using **File** > **Data** > **Review**. To be safe, use **File** > **Data** > **Save . . .** to save your data. Otherwise it will be lost if the program closes. If the program is closed, use **Load . . .** under the same menu to retrieve the saved file.

Print a copy of your data. Select **File, Data, Print, Data Table** and attach the data table to the end of your lab report

Data Analysis

You now need to analyze your data. By plotting position versus time, you will use the data to obtain a graph similar to the one below. (The data shown are for an imaginary moon named CLEA, not one of the moons in the laboratory exercise.)

We know the following: (1) the orbits of the moons are regular, that is, they do not speed up or slow down from one period to the next, and (2) the radius of each orbit does not change from one period to the next. The sine curve should therefore also be regular. It should go through all of the points, and not have a varying maximum height or a varying width from peak to peak.

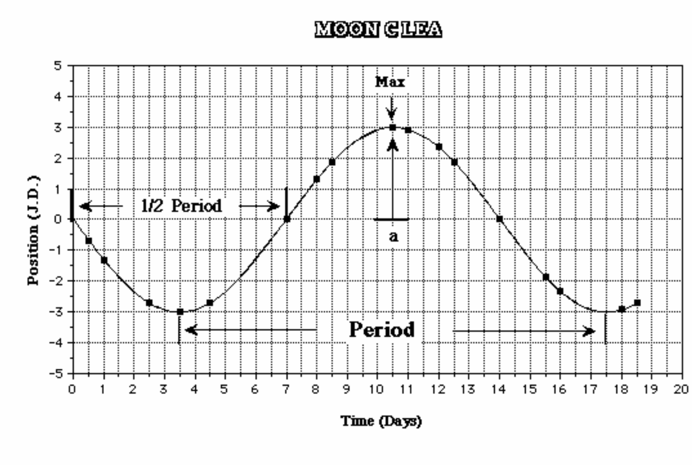

Figure 5

SAMPLE GRAPH FOR MOON CLEA

p = 14 days = 0.0383 years
a = 3 J.D. = 0.00286 A.U.

Taking as an example the imaginary Moon CLEA, we can determine the radius and period of the orbit. The period is the time it takes for the moon to circle the planet and return to the same point in the orbit. Thus the time between two maxima is the period. The time between crossings at 0 Jupiter diameters is equal to half the period because this is the time it takes to go from the front of Jupiter to the back of Jupiter, or half way around. You may find the time between crossings at 0 Jupiter diameters to be of use to you in determining the period, even though the moon has not gone through a complete orbit.

The radius of an orbit is equal to the maximum position eastward or westward, that is, the largest apparent distance from the planet. Remember that the orbits of the moon are nearly circular, but since we see the orbits edge on, we can only determine the radius when the moon is at its maximum position eastward or westward. When you have entered enough data to complete the data sheet you can start analyzing the data.

Fortunately, this program will help you create the appropriate sine curve. Select **File** > **Data** > **Analyze . . .** and then choose your moon from the **Select** menu. When the graph of data points appears, try to find a pattern by eye, and take note of any data points that seem to be out of place. Select **Plot** > **Plot Type** > **Connect Points** to better see the pattern. This will display a jagged, "connect-the-dots" version of your graph. Click on a point at which the line connecting the points crosses ***from negative to positive***. A date and a value very close to zero should appear in the box marked **Cursor Value**. Now select **Plot** > **Fit Sine Curve** > **Set Initial Parameters** and enter the date you just found as **T-zero**. Go back to the graph and find the next point at which the line passes ***from negative to positive*** (that is, when the moon has completed one orbit.) Subtract the earlier date from the later date and enter this as **Period** in the dialog box. Finally, estimate the **Amplitude** of the sine curve by clicking on the peaks and valleys of your graph and reading the **Value** in the **Cursor Position** box. The **Amplitude** is equal to the absolute value of the highest peak or the lowest valley of your graph. When you click on **OK** a screen will appear that allows you to adjust the period and amplitude to best-fit your data.

Write down the adjusted value of the period and amplitude for your moon in data sheet B listed below. Collaborate with other groups to include the best-fit period and amplitude of the other moons listed in the table. Recall that the amplitude is equal to the radius. We are assuming the orbit is circular and that the semi-major axis is equal to the radius.

Data Sheet B

Moon	**Period (days)**	**Period (years)**	**Semi-major Axis (J.D.)**	**Semi-major Axis (A.U.)**
IV Callisto				
III Ganymede				
II Europa				
I Io				

You now have enough information to find the mass of Jupiter. Note that values you obtained from the graphs have units of days for **p,** and Jupiter diameters for **a**. In order to use Kepler's Third Law, you need to convert the period into years and the orbital radius into A.U. Show your conversions from days to years and Jupiter diameters to A.U. in the space below. (1 A.U. = 1050 Jupiter diameters).

Calculating Jupiter's Mass

Calculate a mass of Jupiter using data from each of the four moons. If one of the values differs significantly from the other three, look for a source of error.

$$\mathbf{M} = \frac{\mathbf{a}^3}{\mathbf{p}^2}$$

where $\mathbf{M_J}$ the mass of Jupiter in units of the solar mass

$\mathbf{a}$ is the radius of the orbit in units of A.U.

$\mathbf{p}$ is the period of the orbit in Earth years

From Callisto M_J= ____________________

From Ganymede M_J= ____________________

From Europa M_J= ____________________ **in units of solar masses**

From Io M_J= ____________________

Average M_J = ____ solar masses

Questions

1. Convert the mass of Jupiter into earth masses. (3.00×10^{-6} solar masses = 1 Earth mass) Show your work.

Average M_J = ____earth masses.

2. There are moons beyond the orbit of Callisto. Will they have larger or smaller periods than Callisto? Why?

3. The orbital period of the Earth's moon is 27.3 days and its orbital radius (semi-major axis) is 2.56×10^{-3} A.U. Determine the mass of the Earth in solar masses. Show your work.